hep-th/0407015

N=4 superconformal mechanics in the pp-wave limit

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Abstract

We constructed the pp-wave limit of $N = 4$ superconformal mechanics with the off-shell $(\mathbf{3}, \mathbf{4}, \mathbf{1})$ multiplet. We present the superfield and the component actions which exhibit the interesting property that the interaction parts are completely fixed by the symmetry. We also explicitly demonstrate that the passing to the pp-wave limit can be achieved by keeping at most quadratic nonlinearities in the action of (super)conformal mechanics.

1 Introduction

The simplest example of the famous AdS/CFT correspondence [1], which relates the string theory on $AdS_{p+2}\times S^{D-p-2}$ to extended superconformal theories in $p+1$ dimensions, is provided by the theory of a 0-brane on $AdS_2 \times S^2$. Owing to the AdS/CFT conjecture, the action describing the motion of a super 0-brane in $AdS_2\times S^2$ background should be related to the action of $N=4$ superconformal mechanics with $SU(1,1|2)$ superconformal symmetry. The first step toward establishing this relation was done in [2] where it was shown that the radial motion of a superparticle with zero angular momentum near the horizon of an extreme Reissner-Nordström black hole is described by an $Osp(1|2)$ superconformal mechanics. A proper accounting of the angular degrees of freedom brings us to a new variant of $N=4$ superconformal mechanics [3] containing three physical bosonic fields in its supermultiplet (i.e. the radial AdS_2 coordinate and two angular coordinates parameterizing S^2). A Green-Schwarz-type action for the $AdS_2 \times S^2$ superparticle was constructed in [4]. After a proper gauge-fixing, the corresponding action can be related with $N=4$ superconformal mechanics [5, 6].

Apart from $AdS_{p+2}\times S^{D-p-2}$ and flat space, there exists another maximally supersymmetric background – i.e. the pp-wave background [7]. One of the nice features of the pp-wave background is that string theory can be solved exactly on it [8]. Therefore, it is obviously interesting to understand the role of AdS/CFT duality in the Penrose limit. Usually, the pp-wave limit is considered on the "string" side (see e.g. [9] and references therein), while the "conformal" side is much less understood [10]. In this respect, the $N=4$ superconformal mechanics, being equivalent to a super 0-brane in $AdS_2 \times S^2$ background, provides a nice and simple toy theory to fully understand what happens in the pp-wave limit on the "conformal" side. Our aim in this work is to construct the pp-wave limit of $N=4$ superconformal mechanics.

The most convenient framework for constructing superconformal quantum mechanics with extended supersymmetries is based on nonlinear realizations of $d=1$ superconformal groups. [1] . It was pioneered in [13] and recently advanced in [3, 14, 15]. In the present paper we apply this method to consider nonlinear realizations of the pp-wave limit of the conformal supergroup $SU(1,1|2)$. In this way we re-derive the off-shell multiplet (**3, 4, 1**) [16] which is still useful in the pp-wave limit and construct nontrivial off-shell superfield actions.

The paper is organized as follows. In Section 2 we demonstrate how the nonlinear realizations technique works in the bosonic case by considering $AdS_2 \times S^2$ conformal mechanics in the pp-wave limit. In Section 3 we present $N=4$ superfield formulations of $N=4$ pp-wave superconformal mechanics. The last Section is left for the summary and conclusions.

[1] We recall that superconformal quantum mechanics is also closely related to the integrable Calogero-Moser-type systems [11, 12].

2 Conformal mechanics in the pp-wave limit

2.1 Conformal mechanics on $AdS_2 \times S^2$

We will start with conformal mechanics on $AdS_2 \times S^2$. The symmetry underlying this case is $so(1,2) \oplus su(2)$

$$
\begin{aligned}
&i\,[D,P] = P, \quad i\,[D,K] = -K, \quad i\,[P,K] = -2D && so(1,2) \\
&i\,[V_3,V] = -V, \quad i\left[V_3,\overline{V}\right] = \overline{V}, \quad i\left[V,\overline{V}\right] = 2V_3 && su(2)\,.
\end{aligned}
\tag{2.1}
$$

We will be interested in the nonlinear realization of $SO(1,2)\times SU(2)$ in its coset space $SO(1,2)\times SU(2)/U(1) \sim AdS_2 \times S^2$. There are different choices for the parametrization of this coset (see e.g. [5]) which are actually equivalent, but the simplest one is provided by

$$
g = e^{itP} e^{i\mathcal{Z}K} e^{i\mathcal{U}D} e^{i\Phi V + i\overline{\Phi}\overline{V}}\,. \tag{2.2}
$$

The coset parameters $\left\{\mathcal{U}, \mathcal{Z}, \Phi, \overline{\Phi}\right\}$ are Goldstone fields depending on the time t. In order to construct an invariant action, one should calculate the left-invariant Cartan forms which are defined in a standard way

$$
g^{-1}dg = i\omega_P P + i\omega_D D + i\omega_K K + i\omega_V V + i\bar{\omega}_V \overline{V} + \omega_3 V_3 \tag{2.3}
$$

and explicitly read

$$
\begin{aligned}
&\omega_P = e^{-\mathcal{U}}dt, \quad \omega_D = d\mathcal{U} - 2\mathcal{Z}dt, \quad \omega_K = e^{\mathcal{U}}\left(d\mathcal{Z} + \mathcal{Z}^2 dt\right), \\
&\omega_V = \frac{d\Lambda}{1+\Lambda\overline{\Lambda}}, \quad \bar{\omega}_V = \frac{d\overline{\Lambda}}{1+\Lambda\overline{\Lambda}}, \quad \omega_3 = \frac{\overline{\Lambda}d\Lambda - \Lambda d\overline{\Lambda}}{1+\Lambda\overline{\Lambda}}\,,
\end{aligned}
\tag{2.4}
$$

where

$$
\Lambda = \frac{\tan\sqrt{\Phi\overline{\Phi}}}{\sqrt{\Phi\overline{\Phi}}}\Phi, \quad \overline{\Lambda} = \frac{\tan\sqrt{\Phi\overline{\Phi}}}{\sqrt{\Phi\overline{\Phi}}}\overline{\Phi}\,. \tag{2.5}
$$

The transformation properties of the time t and fields $\left\{\mathcal{U}, \mathcal{Z}, \Phi, \overline{\Phi}\right\}$ under the group $SO(1,2)\times SU(2)$ are generated by the left action of the coset element. For example, the $SU(2)/U(1)$ coset transformations are generated by the left action of the element

$$
g_1 = e^{iaV + i\bar{a}\overline{V}} \tag{2.6}
$$

and read

$$
\delta\Lambda = a + \bar{a}\Lambda^2, \quad \delta\overline{\Lambda} = \bar{a} + a\overline{\Lambda}^2\,. \tag{2.7}
$$

Let us note that the Cartan forms ω_P, ω_D and ω_K are invariant with respect to $SO(1,2)\times SU(2)$ transformations. So we can reduce the number of independent fields using the Inverse Higgs constraint [17]

$$
\omega_D = 0 \quad \Rightarrow \quad \mathcal{Z} = \frac{1}{2}\dot{\mathcal{U}}\,. \tag{2.8}
$$

Now we can write the most general invariant action as

$$
S_{conf} = \kappa \int dt\left(-\omega_K + \alpha m^2 \omega_P + \beta \nabla_t\Lambda\nabla_t\overline{\Lambda}\omega_P + i\gamma m\omega_3\right)\,, \tag{2.9}
$$

where the covariant derivatives of the fields Λ and $\overline{\Lambda}$ are defined as

$$\left\{ \begin{array}{l} \omega_V = \omega_P \nabla_t \Lambda \\ \bar{\omega}_V = \omega_P \nabla_t \overline{\Lambda} \end{array} \right. \quad \Rightarrow \quad \left\{ \begin{array}{l} \nabla_t \Lambda = e^{\mathcal{U}} \frac{\dot{\Lambda}}{1+\Lambda\overline{\Lambda}} \\ \nabla_t \overline{\Lambda} = e^{\mathcal{U}} \frac{\dot{\overline{\Lambda}}}{1+\Lambda\overline{\Lambda}} \end{array} \right. \tag{2.10}$$

The parameters α, β, γ are dimensionless, while κ^{-1} and the constant m have the dimension of mass $[cm^{-1}]$. Explicitly, the action (2.9) reads

$$S_{conf} = \kappa \int dt \left(\frac{1}{4} e^{\mathcal{U}} \dot{\mathcal{U}}^2 + \alpha m^2 e^{-\mathcal{U}} + \beta e^{\mathcal{U}} \frac{\dot{\Lambda}\dot{\overline{\Lambda}}}{(1+\Lambda\overline{\Lambda})^2} + i\gamma m \frac{\overline{\Lambda}\dot{\Lambda} - \Lambda\dot{\overline{\Lambda}}}{1+\Lambda\overline{\Lambda}} \right) . \tag{2.11}$$

This action combines the action of conformal mechanics [18] and that of a charged particle moving in the field of a Dirac monopole. With a specific choice of the values of the arbitrary coefficients α, β and γ, the action (2.11) coincides with the bosonic sector of $N = 4, 8$ superconformal mechanics [3, 15].

2.2 Conformal mechanics in the pp-wave limit

Our strategy is as follows. Firstly, we construct the pp-wave limit for the $so(1,2) \oplus su(2)$ algebra and then repeat all steps to find the invariant action.

In order to find the pp-wave limit of the $so(1,2) \oplus su(2)$ algebra [10], we define the new generators $P_\pm, P_1$, instead of P, K, V_3, as follows:

$$P_\pm = \frac{1}{2}\left(P + m^2 K\right) \pm imV_3 \;, \quad P_1 = \frac{1}{2}\left(K - m^{-2}P\right) \;, \tag{2.12}$$

where the constant m has the same dimension $[cm^{-1}]$ as before. We also make the following rescaling of all generators:

$$P_+ \to \Omega^2 P_+, \; P_- \to P_-, \; \left\{P_1, D, V, \overline{V}\right\} \to \left\{\Omega P_1, \Omega D, \Omega V, \Omega \overline{V}\right\} \; . \tag{2.13}$$

The pp-wave limit corresponds to the limit $\Omega \to 0$ and the algebra (2.1) is reduced, in this limit, to

$$\begin{aligned} & i\left[P_-, D\right] = m^2 P_1, \quad i\left[P_1, D\right] = \frac{1}{2m^2} P_+, \quad i\left[P_1, P_-\right] = D, \\ & i\left[P_-, V\right] = mV, \quad i\left[P_-, \overline{V}\right] = -m\overline{V}, \quad i\left[V, \overline{V}\right] = -\frac{1}{m} P_+ \; . \end{aligned} \tag{2.14}$$

Now, as we did in the previous subsection, we consider a nonlinear realization of the pp-wave group, with the algebra (2.14) in its coset over the central charge P_+, with an element parameterized as

$$g = e^{itP_-} e^{iuD} e^{izP_1} e^{i\phi V + i\bar{\phi}\overline{V}} \; . \tag{2.15}$$

A left shift of the pp-wave group element (2.15) by

$$g_2 = e^{iaP_-} e^{ibD} e^{icP_1} e^{ifV + i\bar{f}\overline{V}} \tag{2.16}$$

induces the following transformations:

$$\delta t = a,\ \delta u = b\cos(mt) + \frac{c}{m}\sin(mt),\ \delta z = -mb\sin(mt) + c\cos(mt),\ \delta\phi = fe^{-imt}\,. \tag{2.17}$$

The left-invariant Cartan forms are given by the following expressions:

$$\begin{aligned}
&\omega_- = dt\,, \quad \omega_D = du - zdt\,, \quad \omega_1 = dz + m^2 u dt\,,\\
&\omega_+ = \frac{1}{4}\left(u^2 + \frac{z^2}{m^2} + 4\phi\bar\phi\right)dt - \frac{1}{2m^2}zdu + \frac{i}{2m}\left(\phi d\bar\phi - \bar\phi d\phi\right)\,,\\
&\omega_V = d\phi + im\,\phi\,dt\,, \bar\omega_V = d\bar\phi - im\,\bar\phi\,dt\,.
\end{aligned} \tag{2.18}$$

Let us note that all coset forms are invariant with respect to (2.17), while ω_+ is shifted by a full differential

$$\delta\omega_+ = d\left[\frac{1}{2m^2}\left(mb\sin(mt) - c\cos(mt)\right)u + \frac{i}{2m}\left(fe^{-imt}\bar\phi - \bar f e^{imt}\phi\right)\right]\,. \tag{2.19}$$

After expressing the field z in terms of u

$$\omega_D = 0 \quad \Rightarrow \quad z = \dot u\,, \tag{2.20}$$

we can write the invariant pp-wave action

$$S_{pp} = \kappa\int dt\left[-m^2\omega_+ + \rho\omega_1 + \sigma\,\nabla_t\phi\nabla_t\bar\phi\,\omega_- + \mu\,m^2\,\omega_- + \nu\,m\,\omega_V + \bar\nu\,m\,\bar\omega_V\right], \tag{2.21}$$

where

$$\nabla_t\phi = \dot\phi + im\,\phi\,, \quad \nabla_t\bar\phi = \dot{\bar\phi} - im\,\bar\phi\,. \tag{2.22}$$

Explicitly, the action (2.21) reads

$$\begin{aligned}
S_{pp} \;=\; &\kappa\int dt\left[\frac{1}{4}\left(\dot u^2 - m^2u^2\right) - m^2\phi\bar\phi + \frac{im}{2}\left(\bar\phi\dot\phi - \phi\dot{\bar\phi}\right) + m^2\rho\,u\right.\\
&\left.+\sigma\left(\dot\phi\dot{\bar\phi} - im\left(\bar\phi\dot\phi - \phi\dot{\bar\phi}\right) + m^2\phi\bar\phi\right) + \mu\,m^2 + i\nu\,m^2\phi - i\bar\nu\,m^2\bar\phi\right]\,.
\end{aligned} \tag{2.23}$$

The action (2.23) is the most general invariant pp-wave action we could construct.

Before comparing this action with the conformal invariant action on $AdS_2\times S^2$ (2.11), we wish to make some comments. First of all, the last two terms in (2.23) cannot be obtained by any reduction procedure from the action (2.11). Their appearance in the pp-wave action (2.23) becomes admissible, due to the reduction of the $U(1)$ symmetry generated by the V_3 generator to the central charge P_+ in the pp-wave algebra (2.14). Thus, in order to compare with the $AdS_2\times S^2$ action, we will put $\nu = 0$. Secondly, we introduce the new fields $\lambda, \bar\lambda$

$$\lambda = e^{imt}\phi, \quad \bar\lambda = e^{-imt}\bar\phi \tag{2.24}$$

and rewrite the action (2.23) (with $\nu = 0$) as

$$S_{pp} = \kappa\int dt\left[\frac{1}{4}\left(\dot u^2 - m^2u^2\right) + \frac{im}{2}\left(\bar\lambda\dot\lambda - \lambda\dot{\bar\lambda}\right) + \rho\,m^2u + \sigma\dot\lambda\dot{\bar\lambda} + \mu\,m^2\right]\,. \tag{2.25}$$

Let us stress that in the pp-wave action (2.25) the mass term for the field u and the WZW term come, together with the kinetic term for u, from the same Cartan forms ω_-. As a result, the mass of field u and the coupling constant in front of the WZW term are completely fixed.

Now, comparing the pp-wave action (2.25) with the $AdS_2 \times S^2$ action (2.11), one may conclude that all terms in the pp-wave action can be obtained from the $AdS_2 \times S^2$ one by keeping there only terms at most quadratic in the fields. Additionally, in order to have exact matching, we have to restrict the arbitrary coefficients, which are present in both actions, as follows:

$$\alpha = -\frac{1}{2},\ \gamma = \frac{1}{2},\ \rho = \frac{1}{2},\ \sigma = \beta,\ \mu = -\frac{1}{2}. \tag{2.26}$$

Thus, we conclude that the net effect of taking the pp-wave limit in conformal mechanics consists in reducing all nonlinearities to quadratic ones, together with fixing the value of some otherwise arbitrary coefficients in the action. Let us note, however, that such a fixing of the coefficients is not really needed. The actions (2.11) and (2.25) are invariant with respect to $so(1,2) \oplus su(2)$ and pp-wave algebra (2.14) transformations respectively, without any fixing of the coefficients. The relations between coefficients appear only if we wish to get the pp-wave action from $AdS_2 \times S^2$ one.

So, in the pp-wave limit the theories on both sides, i.e. the "string/particle" and the conformal mechanics side, contain at most quadratic interaction terms, which represent indeed mass terms for some fields and WZW terms. In the next section we will demonstrate that the same conclusion is still valid in the supersymmetric case.

3 N=4 superconformal mechanics in the pp-wave limit

The construction of $N = 4$ superconformal mechanics in the pp-wave limit is similar to the general consideration of $N = 4$ superconformal mechanics, which can be found in [3]. Precisely, we will start with the superconformal algebra $su(1,1|2)$ and pass to its pp-wave limit in two steps[2]

- Firstly, we redefine the bosonic generator as in (2.12) and the spinor generators as

$$\widetilde{Q}^1 = \frac{1}{2}\left(Q^1 - imS^1\right), \widetilde{Q}^2 = \frac{1}{2}\left(Q^2 + imS^2\right),$$
$$\widetilde{S}^1 = \frac{1}{2m}\left(Q^1 + imS^1\right), \widetilde{S}^2 = \frac{1}{2m}\left(Q^2 - imS^2\right) \tag{3.1}$$

- Secondly, we rescale the bosonic generators as in (2.13) and the spinor ones as

$$\widetilde{Q}^i \to \widetilde{Q}^i\ , \quad \widetilde{S}^i \to \Omega\widetilde{S}^i \tag{3.2}$$

  and consider the limit $\Omega \to 0$.

[2] We use the notation of [3].

Finally, we get the following pp-wave superalgebra (together with (2.14)):

$$\left[P_-,\widetilde{S}^1\right]=-m\widetilde{S}^1,\ \left[P_-,\widetilde{S}^2\right]=m\widetilde{S}^2,\ \left[P_1,\widetilde{Q}^1\right]=-\frac{1}{2}\widetilde{S}^1,\ \left[P_1,\widetilde{Q}^2\right]=\frac{1}{2}\widetilde{S}^2,$$
$$\left[D,\widetilde{Q}^i\right]=-\frac{im}{2}\widetilde{S}^i,\ \left[V,\widetilde{Q}^1\right]=-im\widetilde{S}^2,\ \left[\overline{V},\widetilde{Q}^2\right]=im\widetilde{S}^1,\left\{\widetilde{Q}^1,\widetilde{\overline{S}}_2\right\}=-i\overline{V},$$
$$\left\{\widetilde{Q}^2,\widetilde{\overline{S}}_1\right\}=-iV,\ \left\{\widetilde{Q}^1,\widetilde{\overline{S}}_1\right\}=iD+mP_1,\ \left\{\widetilde{Q}^2,\widetilde{\overline{S}}_2\right\}=-iD+mP_1,$$
$$\left\{\widetilde{Q}^i,\widetilde{\overline{Q}}_j\right\}=-\delta^i_jP_-,\ \left\{\widetilde{S}^i,\widetilde{\overline{S}}_j\right\}=-m^{-2}\delta^i_jP_+. \tag{3.3}$$

Next, we shall construct a nonlinear realization of the pp-wave supergroup with superalgebra (2.14), (3.3) on the coset superspace parameterized as

$$g=e^{itP_-}e^{\theta_i\widetilde{Q}^i+\bar\theta^i\widetilde{\overline{Q}}_i}e^{\psi_i\widetilde{S}^i+\bar\psi^i\widetilde{\overline{S}}_i}e^{iuD}e^{izP_1}e^{i\phi V+i\bar\phi\overline{V}}\ . \tag{3.4}$$

Now, the coordinates $t,\theta_i,\bar\theta^i$ parameterize the $N=4,d=1$ superspace, while the rest of the coset parameters are Goldstone superfields.

The transformation properties of the coordinates and superfields are generated by acting on the coset element (3.4) from the left with the elements of the pp-wave group. The $N=4$ super Poincaré transformations are generated by the element

$$g_0=\exp\left(\varepsilon_1\widetilde{Q}^i+\bar\varepsilon^i\widetilde{\overline{Q}}_i\right)\ . \tag{3.5}$$

They read

$$\delta t=-\frac{i}{2}\left(\varepsilon_i\bar\theta^i+\bar\varepsilon^i\theta_i\right),\ \delta\theta_i=\varepsilon_i,\ \delta\bar\theta^i=\bar\varepsilon^i \tag{3.6}$$

and all superfields are scalars.

The transformations under S-supersymmetry are generated by the element

$$g_1=\exp\left(\epsilon_1\widetilde{S}^i+\bar\epsilon^i\widetilde{\overline{S}}_i\right) \tag{3.7}$$

and have the following explicit form:

$$\delta u=-\bar\theta^1\tilde\epsilon_1+\bar\theta^2\tilde\epsilon_2+\theta_1\bar{\tilde\epsilon}^1-\theta_2\bar{\tilde\epsilon}^2,\ \delta z=-im\left(\bar\theta^i\tilde\epsilon_i+\theta_i\bar{\tilde\epsilon}^i\right),$$
$$\delta\phi=\bar\theta^1\tilde\epsilon_2-\theta_2\bar{\tilde\epsilon}^1,\ \delta\bar\phi=\bar\theta^2\tilde\epsilon_1-\theta_1\bar{\tilde\epsilon}^2,\ \delta\psi_1=\tilde\epsilon_1-m\theta_1\theta_2\bar{\tilde\epsilon}^2,\ \delta\psi_2=\tilde\epsilon_2-m\theta_1\theta_2\bar{\tilde\epsilon}^1, \tag{3.8}$$

where

$$\tilde\epsilon_1=\epsilon_1e^{im\tilde t},\ \tilde\epsilon_2=\epsilon_2e^{-im\tilde t},\ \tilde t=t+\frac{1}{2}\theta_i\bar\theta^i\ . \tag{3.9}$$

Since all other super pp-wave group transformations appear in the anticommutators of Poincaré and S supersymmetries, it is sufficient, when constructing the action, to require invariance under these two supersymmetries.

In what follows we will need the explicit structure of several important Cartan forms, defined in a standard way as $g^{-1}dg$

$$\omega_-=dt+\frac{i}{2}\left(\theta_id\bar\theta^i+\bar\theta^id\theta_i\right)\equiv\triangle t,$$
$$\omega_D=du-z\triangle t-\psi_1d\bar\theta^1+\psi_2d\bar\theta^2+\bar\psi^1d\theta_1-\bar\psi^2d\theta_2,$$
$$\omega_V=d\phi+im\phi\triangle t+\psi_2d\bar\theta^1-\bar\psi^1d\theta_2,\ \bar\omega_V=d\bar\phi-im\bar\phi\triangle t-\bar\psi^2d\theta_1+\psi_1d\bar\theta^2. \tag{3.10}$$

Now we may define covariant spinor derivatives as

$$D^i = \frac{\partial}{\partial\theta_i} + \frac{i}{2}\bar\theta^i\partial_t,\ \overline{D}_i = \frac{\partial}{\partial\bar\theta^i} + \frac{i}{2}\theta_i\partial_t,\ \left\{D^i, \overline{D}_j\right\} = i\delta^i_j\partial_t\ . \tag{3.11}$$

Prior to constructing the invariant action, one should impose proper irreducibility constraints on the $N=4$ superfields. The basic idea for finding the appropriate constraints is the same as in the case of superconformal mechanics [3]: we impose constraints such that our basic, low-dimensional bosonic superfields $u, \phi, \bar\phi$ contain among their components only four fermions, which should coincide with the first components of the spinor superfields $\psi_i, \bar\psi^i$. These invariant conditions represent a particular case of the Inverse Higgs effect [17] and can be written as

$$\omega_D = 0\ ,\quad \omega_V\,| = 0\ ,\quad \bar\omega_V| = 0\ , \tag{3.12}$$

where $|$ denotes $d\theta$ and $d\bar\theta$ projections of the forms. Explicitly, the constraints (3.12) read

$$\begin{aligned} &D^1 u = -D^2\phi = \bar\psi^1,\ D^2 u = D^1\bar\phi = -\bar\psi^2\ ,\ D^1\phi = \overline{D}_2\phi = 0, \\ &\overline{D}_1 u = -\overline{D}_2\bar\phi = -\psi_1,\ \overline{D}_2 u = \overline{D}_1\phi = \psi_2\ ,\ D^2\bar\phi = \overline{D}_1\bar\phi = 0. \end{aligned} \tag{3.13}$$

After introducing a new $N=4$ "vector" superfield V^{ij} via

$$V^{11} = i\sqrt{2}\,\phi,\ V^{22} = -i\sqrt{2}\,\bar\phi,\ V^{12} = \frac{i}{\sqrt{2}}u,\quad \overline{V^{ij}} = V_{ij} \tag{3.14}$$

the constraints (3.13) can be brought in the familiar form

$$D^{(i}V^{jk)} = 0\ ,\quad \overline{D}^{(i}V^{jk)} = 0\ . \tag{3.15}$$

The superfield V^{ij} subject to (3.15) is recognized as the $N=4, d=1$ tensor multiplet [16, 3] with (**3,4,1**) off-shell components content. Of course, in the present case the explicit $su(2)$ invariant form of constraints is a fake, because in the pp-wave limit the $su(2)$ symmetry is broken down to the algebra

$$i\left[V, \bar V\right] = -m^{-1}\,P_+ \tag{3.16}$$

with P_+ being a central charge. Let us note that it is quite natural to have the same $N=4$ tensor multiplet as in the case of the $N=4$ superconformal algebra [3], also in the case of its pp-wave limit. The reason for this is evident: as we showed in the bosonic case in the previous Section, passing to the pp-wave limit means keeping only at most quadratic nonlinearities in the Lagrangian, provided that the fields have been chosen properly. Now, in the supersymmetric case we see that constraints linear in the superfields are preserved in the pp-wave limit. Thus, it is natural to suggest that the invariant superfields Lagrangian should be related with the full $N=4$ superconformal one in the same way as in the purely bosonic case. Now we are going to demonstrate that this is really so.

The invariant superfield action consists of a superfield kinetic term and a superpotential. As in the $N=4$ superconformal mechanics case [3], the kinetic and potential terms are easier to write in $N=2$ superspace, let us say the one with coordinates $\left\{t, \theta_2, \bar\theta^2\right\}$. The analysis of the constraints (3.13) shows that in the $\theta_1, \bar\theta^1$ expansion of the $N=4$ superfields $u, \phi, \bar\phi$, only the $\theta_1 = \bar\theta^1 = 0$ components of each superfield are independent $N=2$ superfields [3]. Let us denote these superfields as

$$u| = v,\ \phi\,| = \rho,\ \bar\phi| = \bar\rho,\quad D^2\bar\rho = \overline{D}_2\rho = 0\ , \tag{3.17}$$

where | indicates the $\theta_1 = \bar{\theta}^1 = 0$ restriction.

The transformations of the implicit $N=2$ Poincaré supersymmetry generated by $\widetilde{Q}_1, \widetilde{\overline{Q}}{}^1$ have the following form, in terms of these $N=2$ superfields:

$$\delta v = \varepsilon_1 D^2 \rho + \bar{\varepsilon}^1 \overline{D}_2 \bar{\rho} \ , \ \delta \rho = -\bar{\varepsilon}^1 \overline{D}_2 v \ , \ \delta \bar{\rho} = -\varepsilon_1 D^2 v \ , \tag{3.18}$$

while under $\widetilde{S}$-supersymmetries they transform as

$$\delta v = -\epsilon_2 \bar{\theta}^2 e^{-imt} + \bar{\epsilon}^2 \theta_2 e^{imt}, \quad \delta \rho = \bar{\epsilon}^1 \theta_2 e^{-imt}, \ \delta \bar{\rho} = -\epsilon_1 \bar{\theta}^2 e^{imt} \ . \tag{3.19}$$

The kinetic and potential terms which are independently invariant with respect to the implicit $N=2$ Poincaré supersymmetry (3.18) can be easily found to be

$$S_{kin} = A \int dt d^2\theta_2 \left(D^2 v \overline{D}_2 v + D^2 \rho \overline{D}_2 \bar{\rho} \right) , \tag{3.20}$$

$$S_{pot} = B \int dt d^2\theta_2 \left(v^2 - 2\rho\bar{\rho} \right) . \tag{3.21}$$

So, the sum of the actions (3.20) and (3.21) possesses $N=4$ super Poincaré supersymmetry. Finally, one should check the invariance of the actions with respect to (3.19). After doing so, one can conclude that only the sum of S_{kin} and S_{pot} possesses this symmetry, provided that

$$2B = mA \ . \tag{3.22}$$

Thus, the invariant action of $N=4$ superconformal mechanics in pp-wave limit has the following form (with $A=\kappa$):

$$S_{s-pp} = \kappa \int dt d^2\theta_2 \left[D^2 v \overline{D}_2 v + D^2 \rho \overline{D}_2 \bar{\rho} + \frac{m}{2} \left(v^2 - 2\rho\bar{\rho} \right) \right] \ , \tag{3.23}$$

without *any arbitrary coefficients* besides m. The invariance with respect to pp-wave supergroup strictly fixes all possible interacting terms.

It is instructive to rewrite the action (3.23) in terms of components fields, which can be defined as

$$v|, \ \psi = i D^2 v|, \ \bar{\psi} = i \overline{D}_2 v|, \ \mathcal{A} = \left[D^2, \overline{D}_2 \right] v|, \quad \xi = D^2 \rho|, \ \bar{\xi} = \overline{D}_2 \rho| \ , \tag{3.24}$$

where | means $\theta_2 = \bar{\theta}^2 = 0$. Integrating over θ in (3.23) and eliminating the auxiliary field $\mathcal{A}$ by making use of its equation of motion, we end up with the following action:

$$S_{s-pp} = \kappa \int dt \left[\frac{1}{4} \dot{v}^2 - \frac{m^2}{4} v^2 + \dot{\rho}\dot{\bar{\rho}} + \frac{im}{2} \left(\rho \dot{\bar{\rho}} - \dot{\rho} \bar{\rho} \right) + i \dot{\psi} \bar{\psi} + i \dot{\xi} \bar{\xi} + m \left(\psi \bar{\psi} - \xi \bar{\xi} \right) \right] . \tag{3.25}$$

Finally, one can conclude that in the fully supersymmetric case the pp-wave action represents a limiting case of the $N=4$ superconformal mechanics action, where only nonlinearities at most quadratic in the superfields survive.

Conclusion

Motivated by the interest in understanding the role of AdS/CFT duality in the Penrose limit, whose "conformal" side has received so far much less attention than the corresponding "string" side, we constructed in this paper the pp-wave limit of $N = 4$ superconformal mechanics with the off-shell $(\mathbf{3}, \mathbf{4}, \mathbf{1})$ multiplet. We showed that this multiplet can be described by a properly constrained Goldstone superfield, associated with a suitable coset of the nonlinearly realized $N = 4, d = 1$ pp-wave supergroup. We presented the superfield and the component actions, which exhibit the interesting property that the interaction part is completely fixed by symmetry. Moreover, for the pp-wave case, the kinetic and potential terms are invariant only when taken together, as a linear combination of the two terms, provided the value if their relative coefficient is appropriately set. We also explicitly demonstrated that the passing to pp-wave limit can be achieved by keeping at most quadratic nonlinearities in the action of (super)conformal mechanics.

Acknowledgments

This work was partially supported by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime, the INTAS-00-00254 grant, the NATO Collaborative Linkage Grant PST.CLG.979389, RFBR-DFG grant No 02-02-04002, grant DFG No 436 RUS 113/669, RFBR grant No 03-02-17440 and a grant of the Heisenberg-Landau programme.

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231, `hep-th/9711200`;
S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. **B428** (1998) 105, `hep-th/9802109`;
E. Witten, Theor. Math. Phys. **2** (1998) 109, `hep-th/9802150`.

[2] P. Claus, M. Derix, R. Kallosh, J. Kumar, P.K. Townsend, A. van Proeyen, Phys. Rev. Lett. **81** (1998) 4553, `hep-th/9804177`.

[3] E. Ivanov, S. Krivonos. O. Lechtenfeld, JHEP **0303** (2003) 014, `hep-th/0212303`.

[4] Jien-Ge Zhou, Nucl. Phys. **B559** (1999) 92, `hep-th/9906013`;
M. Kreuzer, Jien-Ge Zhou, Phys. Lett. **B472** (2000) 309, `hep-th/9910067`.

[5] E. Ivanov, S. Krivonos, J. Niederle, Nucl. Phys. **B677** (2004) 485, `hep-th/0210196`.

[6] S. Bellucci, A. Galajinsky, E. Ivanov, S. Krivonos, Phys. Lett. **B555** (2003) 99, `hep-th/0212204`.

[7] M. Blau, J. Figueroa-O'Farrill, C. Hull, G. Papadopoulos, Class. Quant. Grav. **19** (2002) L87, `hep-th/0110243`.

[8] R.R. Metsaev, Nucl. Phys. **B625** (2002) 70, `hep-th/0112044`;
R.R. Metsaev, A.A. Tseytlin, Phys. Rev. **D65** (2002), `hep-th/0202109`.

[9] R. Russo, A. Tanzini, *"The Duality between IIB String Theory on PP-wave and N=4 SYM: a Status Report"*, Class. Quant. Grav. **21** (2004) S1265, `hep-th/0401155`.

[10] G. Arutyunov, E. Sokatchev, JHEP **0208** (2002) 014, `hep-th/0205270`.

[11] F. Calogero, J. Math. Phys. 10 (1969) 2197.

[12] S. Bellucci, A. Galajinsky, S. Krivonos, Phys. Rev. D68 (2003) 064010, `hep-th/0304087`.

[13] E. Ivanov, S. Krivonos, V. Leviant, J. Phys. **A**: Math. Gen. **22** (1989) 4201.

[14] E. Ivanov, S. Krivonos, O. Lechtenfeld, Class. Quant. Grav. **21** (2004) 1031, `hep-th/0310299`.

[15] S. Bellucci, E. Ivanov, S. Krivonos, O. Lechtenfeld, Nucl. Phys. **B684** (2004) 321, `hep-th/0312322`;
S. Bellucci, E. Ivanov, S. Krivonos, O. Lechtenfeld, *"ABC of N=8, d=1 supermultiplets"*, `hep-th/0406015`.

[16] E.A. Ivanov, A.V. Smilga, Phys. Lett. **B257** (1991) 79;
V.P. Berezovoj, A.I. Pashnev, Class. Quant. Grav. **8** (1991) 2141;
A. Maloney, M. Spradlin, A. Strominger, JHEP **0204** (2002) 003, `hep-th/9911001`.

[17] E.A. Ivanov, V.I. Ogievetsky, Teor. Mat. Fiz. **25** (1975) 164.

[18] V. De Alfaro, S. Fubini, G. Furlan, Nuovo Cim. **A 34** (1974) 569.